

Rationality

Lecture 13

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Game Theory

“We wish to find the mathematically complete principles which define ‘rational behavior’ for the participants.” (pg. 31)

J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, 1944.

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M. Osborne and A. Rubinstein. *Introduction to Game Theory*. MIT Press, 2004.

Rationality in Interaction

What does it mean to be rational when the outcome of one's action depends upon the actions of other people and everyone is trying to guess what the others will do?

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*In social interaction, rationality has to be enriched with further assumptions about individuals' **mutual knowledge and beliefs**, but these assumptions are not without consequence.*

C. Bicchieri. *Rationality and Game Theory*. Chapter 10 in [HR].

Game Situations

1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

	Bob	
	L	R
	1	0

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Game Situations

		Bob	
		<i>L</i>	<i>R</i>
Ann	<i>U</i>	1 1	0 0
	<i>D</i>	0 0	1 1

1. a **group** of *self-interested* agents (players) involved in some interdependent **decision problem**

Game Situations

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		<i>L</i>	<i>R</i>
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		Bob	
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What should Ann (Bob) *do*?

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Who is game theory about?

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1. **Classical view:** idealized world with *perfectly rational agents*
2. **Humanistic view:** real people in interactive situations

L. Samuelson. *Comments on Game Theory*. Game Theory: 5 Questions, Automatic Press, 2007.

Who is game theory about?

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- The game itself is taken to be a literal description of the strategic interaction

"We adhere to the classical point of view that the game under consideration fully describes the real situation — that any (pre) commitment possibilities, any repetitive aspect, any probabilities of error, or any possibility of jointly observing some random event, have already been modeled in the game tree." (pg. 1005)

E. Kohlberg and J.-F. Mertens. *On the strategic stability of equilibria*. *Econometrica*, 54, pgs. 1003 - 1038, 1986.

L. Samuelson. *Comments on Game Theory*. Game Theory: 5 Questions, Automatic Press, 2007.

Who is game theory about?

1. **Classical view:** idealized world with *perfectly rational agents*
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 - Any appropriate concept of equilibrium should be an *implication* of the information provided in the modeled interpreted through an assumption of perfect rationality.
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2. **Humanistic view:** real people in interactive situations

- the mathematical structures are *models* of interactive situations
- the appropriate notion of equilibrium is part of the specification of the model

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What does it mean for Ann to be *perfectly rational*?

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	1,1

What does it mean for Ann to be *perfectly rational*?

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	1,1

Ann's best choice depends on what she *expects* Bob to do, and this depends on what she *thinks* Bob expects her to do, and so on...

Just Enough Game Theory

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A **game** is a description of strategic interaction that includes

- ▶ actions the players *can* take
- ▶ description of the players' interests (i.e., preferences),
- ▶ description of the “structure” of the decision problem

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- ▶ actions the players *can* take
- ▶ description of the players' interests (i.e., preferences),
- ▶ description of the “structure” of the decision problem

It does not specify the actions that the players do take.

A **solution concept** is a systematic description of the outcomes that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards inductions, or iterated dominance of various kinds.

These are usually thought of as the embodiment of “rational behavior” in some way and used to analyze game situations.

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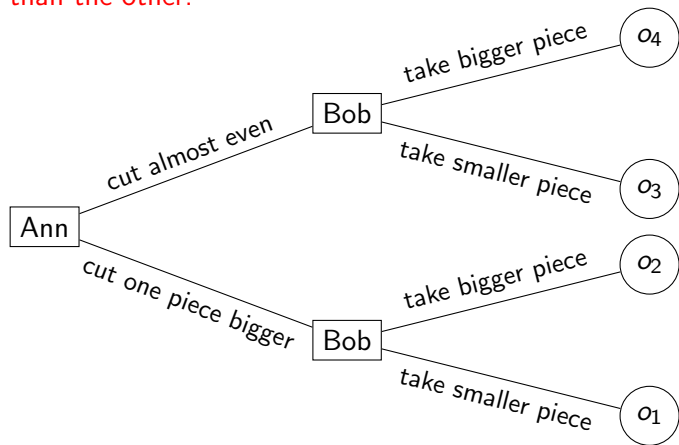
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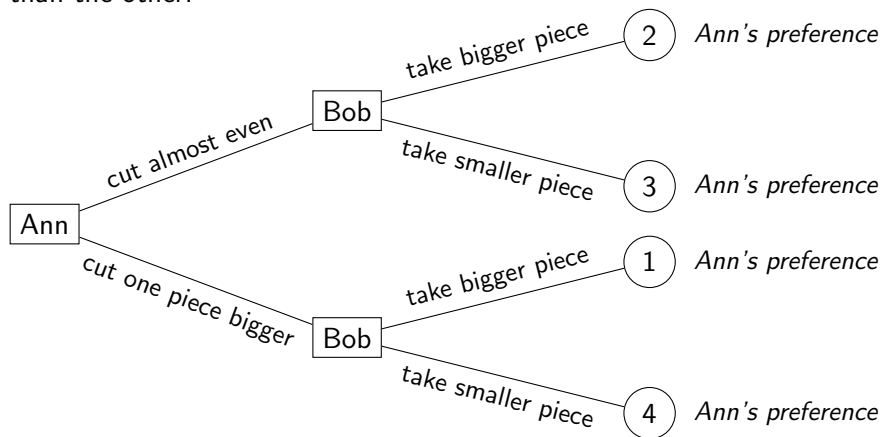
For this course, **solution concepts** are more of an *endpoint*.

Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they *only* care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.

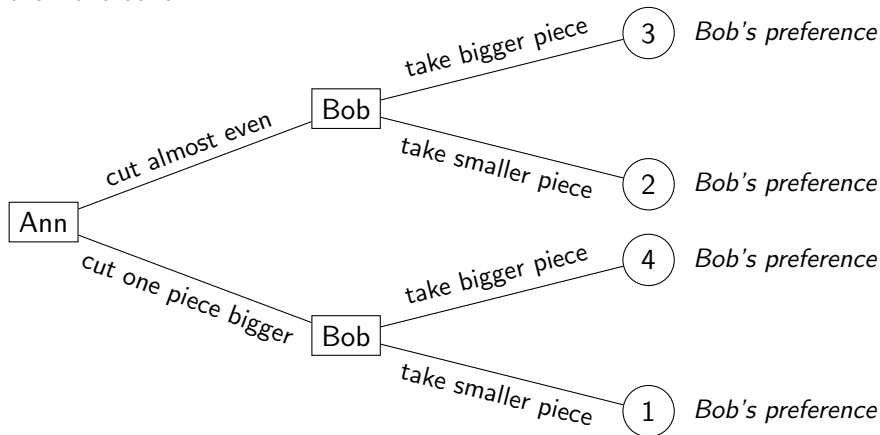
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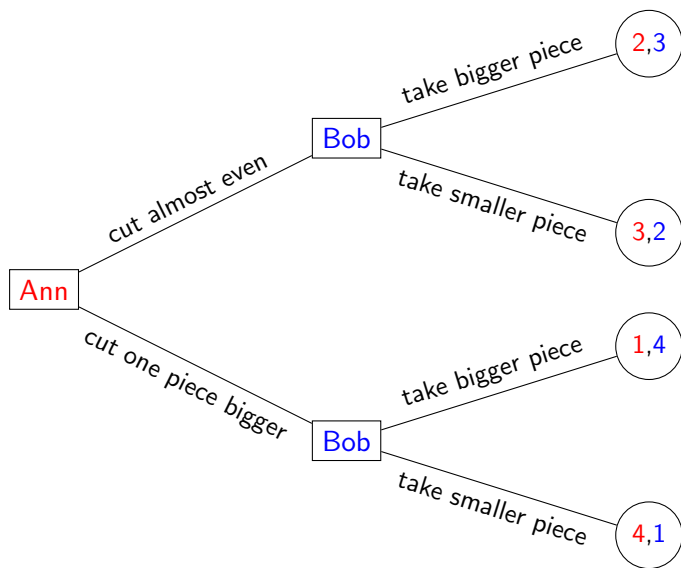


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		Bob	
		TB	TS
Ann	CB	1,4	4,1
	CE	2,3	3,2

What should Ann *do*?

		Bob	
		TB	TS
Ann	CB	1,4	4,1
	CE	2,3	3,2

What should Ann *do*? *Bob's best choice in Ann's worst choice*

		Bob		
		TB	TS	
Ann	CB	1,4	4,1	1
	CE	2,3	3,2	2

What should Ann *do*? *maximize over each row and choose the maximum value*

		Bob	
		<i>TB</i>	<i>TS</i>
Ann	<i>CB</i>	1,4	4,1
	<i>CE</i>	2,3	3,2
		3	1

What should Bob *do*? *minimize over each column and choose the maximum value*

Zero-Sum Games

Von Neumann Minmax Theorem. In any finite, two-player, zero-sum game, there is always at least one minmax solution.

What does it mean for Ann to be *perfectly rational*?

		Bob	
		H	T
Ann	H	1,-1	-1,1
	T	-1,1	1,-1

What is a rational choice for Ann (Bob)?

What does it mean for Ann to be *perfectly rational*?

		Bob	
		H	T
Ann	H	1,-1	-1,1
	T	-1,1	1,-1

What is a rational choice for Ann (Bob)? *Flip a coin!*

What does it mean for Ann to be *perfectly rational*?

		Bob	
		C1	C2
Ann	P1	1,-1	-1,1
	P2	-1,1	1,-1

What is a rational choice for Ann (Bob)?

What does it mean for Ann to be *perfectly rational*?

		Bob	
		C1	C2
Ann	P1	1,-1	-1,1
	P2	-1,1	1,-1

		Bob	
		C1	C2
Ann	P1	1,-1	1,-1
	P2	1,-1	1,-1

What is a rational choice for Ann (Bob)? *Play a different game!*

Strategic Games

A **strategic games** is a tuple $\langle N, \{A_i\}_{i \in N}, \{\succeq_i\}_{i \in N} \rangle$ where

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A **strategic games** is a tuple $\langle N, \{A_i\}_{i \in N}, \{\succeq_i\}_{i \in N} \rangle$ where

- ▶ N is a finite set of **players**
- ▶ for each $i \in N$, A_i is a nonempty set of **actions**
- ▶ for each $i \in N$, $\succeq_i$ is a **preference relation** on $A = \prod_{i \in N} A_i$
(Often $\succeq_i$ are represented by **utility functions** $u_i : A \rightarrow \mathbb{R}$)

Strategic Games: Comments on Preferences

- ▶ Preferences may be over a set of consequences C . Assume $g : A \rightarrow C$ and $\{\succeq_i^* \mid i \in N\}$ a set of preferences on C . Then for $a, b \in A$,

$$a \succeq_i b \text{ iff } g(a) \succeq_i^* g(b)$$

- ▶ Consequences may be affected by exogenous random variable whose realization is not known before choosing actions. Let Ω be a set of states, then define $g : A \times \Omega \rightarrow C$. Where $g(a|\cdot)$ is interpreted as a *lottery*.
- ▶ Often $\succeq_i$ are represented by **utility functions** $u_i : A \rightarrow \mathbb{R}$

Strategic Games: Example

		Column	
		r	l
Row	u		
	d		

- ▶ $N = \{Row, Column\}$
- ▶ $A_{Row} = \{u, d\}$, $A_{Column} = \{r, l\}$
- ▶ $(u, r) \succeq_{Row} (d, l) \succeq_{Row} (u, l) \sim_{Row} (d, r)$
 $(u, r) \succeq_{Column} (d, l) \succeq_{Column} (u, l) \sim_{Column} (d, r)$

Strategic Games: Example

		Column	
		r	l
Row	u	(2,2)	(0,0)
	d	(0,0)	(1,1)

- ▶ $N = \{Row, Column\}$
- ▶ $A_{Row} = \{u, d\}$, $A_{Column} = \{r, l\}$
- ▶ $u_{Row} : A_{Row} \times A_{Column} \rightarrow \{0, 1, 2\}$,
 $u_{Column} : A_{Row} \times A_{Column} \rightarrow \{0, 1, 2\}$ with
 $u_{Row}(u, r) = u_{Column}(u, r) = 2$,
 $u_{Row}(d, l) = u_{Column}(d, l) = 2$,
and $u_x(u, l) = u_x(d, r) = 0$ for $x \in N$.

Nash Equilibrium

Let $\langle N, \{A_i\}_{i \in N}, \{\succeq_i\}_{i \in N} \rangle$ be a strategic game

For $a_{-i} \in A_{-i}$, let

$$B_i(a_{-i}) = \{a_i \in A_i \mid (a_{-i}, a_i) \succeq_i (a_{-i}, a'_i) \ \forall \ a'_i \in A_i\}$$

B_i is the **best-response** function.

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B_i is the **best-response** function.

$a^* \in A$ is a **Nash equilibrium** iff $a_i^* \in B_i(a_{-i}^*)$ for all $i \in N$.

Strategic Games Example: Bach or Stravinsky?

	b_c	s_c
b_r	2,1	0,0
s_r	0,0	1,2

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$$B_r(b_c) = \{b_r\}$$

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$$B_r(s_c) = \{s_r\}$$

$$B_c(b_r) = \{b_c\}$$

$$B_c(s_r) = \{s_c\}$$

(b_r, b_c) is a Nash Equilibrium

(s_r, s_c) is a Nash Equilibrium

Another Example: Pure Coordination

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	1,1

Another Example: Hi-Low

		Bob	
		L	R
Ann	U	3,3	0,0
	D	0,0	1,1

Teamwork, Focal Points

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Question: can teamwork do better than that?

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Intuitively, **Yes**.

“There are these two broad empirical facts about Hi-Lo games, people almost always choose A [Hi] and people with common knowledge of each other’s rationality think it is obviously rational to choose A [Hi].”

[Bacharach, Beyond Individual Choice, 2006, pg. 42]

See also chapter 2 of:

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But then more machinery is needed...

Prisoner's Dilemma

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Prisoner's Dilemma

Two options: Confess (C), Don't Confess (D)

Prisoner's Dilemma

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Possible outcomes:

Prisoner's Dilemma

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Prisoner's Dilemma

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Possible outcomes: We both confess (C, C), I confess but my partner doesn't (C, D),

Prisoner's Dilemma

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Prisoner's Dilemma

Two options: Confess (C), Don't Confess (D)

Possible outcomes: We both confess (C, C), I confess but my partner doesn't (C, D), My partner confesses but I don't (D, C), neither of us confess (D, D).

Prisoner's Dilemma

		Bob	
		<i>D</i>	<i>C</i>
Ann	<i>D</i>		
	<i>C</i>		

Prisoner's Dilemma

		Bob	
		<i>D</i>	<i>C</i>
Ann	<i>D</i>	3	1
	<i>C</i>	4	2

Ann's preferences

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3	4
	C	1	2

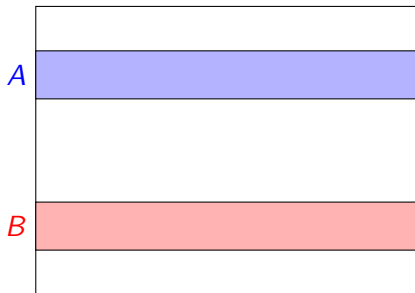
Bob's preferences

Prisoner's Dilemma

		Bob	
		<i>D</i>	<i>C</i>
Ann	<i>D</i>	3,3	1,4
	<i>C</i>	4,1	2,2

What should Ann (Bob) do?

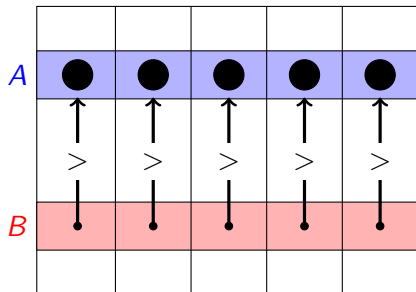
Dominance Reasoning



Dominance Reasoning

<i>A</i>	●	●	●	●
<i>B</i>	●	●	●	●

Dominance Reasoning



Prisoner's Dilemma

		Bob	
		<i>D</i>	<i>C</i>
Ann	<i>D</i>	3,3	1,4
	<i>C</i>	4,1	2,2

What should Ann (Bob) do?

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma

		Bob	
		<i>D</i>	<i>C</i>
Ann	<i>D</i>	3, 3	1, 4
	<i>C</i>	4, 1	2, 2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3, 3	1, 4
	C	4, 1	2, 2

What should Ann (Bob) do? *Dominance reasoning* is not **Pareto**!

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	3	2.5
	C	2.5	2

What should Ann (Bob) do? *Think as a group!*

Prisoner's Dilemma

		Bob	
		<i>D</i>	<i>C</i>
Ann	<i>D</i>	3,3	1,4
	<i>C</i>	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image!*

Prisoner's Dilemma

		Bob	
		<i>D</i>	<i>C</i>
Ann	<i>D</i>	3,3	1,4
	<i>C</i>	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image!*

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	€,€	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Change the game* (eg., Symbolic Utilities)

Next Week: More game theory (common knowledge of rationality, backwards induction), group preferences