Part 2: Gödel’s Proof of the Existence of God

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Review: Intensional vs. Extensional Objects

Extensional Object: a set or relation in the usual sense

Intensional Object: (or concept), the “meaning” depends on the context (i.e., possible world), a function from possible worlds to extensional objects.
**Review: Intensional vs. Extensional Objects**

**Extensional Object:** a set or relation in the usual sense

**Intensional Object:** (or *concept*), the “meaning” depends on the context (i.e., possible world), a function from possible worlds to extensional objects.

*Example:*

- Possible worlds are people, the domain as real-world objects
- Each person will classify some of those objects as being *red* (type $\langle 0 \rangle$).
- The *red concept* maps to each person the set of objects he/she considers red (type $\uparrow \langle 0 \rangle$).
- The *color concept* maps to each person the set of *color* (concepts) for that person (type $\uparrow \uparrow \langle 0 \rangle$).
Someday everybody will be tall

Many ambiguities!
Someday everybody will be tall

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Let $T(x)$ be a (non-fuzzy) predicate saying "$x$ is tall", assume worlds are points in time ($\Diamond \varphi$ means "$\varphi$ will be true"), assume actualist reading for now:
Someday everybody will be tall

Many ambiguities!

Let $T(x)$ be a (non-fuzzy) predicate saying “$x$ is tall”, assume worlds are points in time ($\Diamond \varphi$ means “$\varphi$ will be true”), assume actualist reading for now:

1. $\forall x \Diamond T(x)$
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1. $\forall x \Diamond T(x)$
2. $\Diamond \forall x T(x)$
Someday everybody will be tall

Many ambiguities!

Let $T(x)$ be a (non-fuzzy) predicate saying “$x$ is tall”, assume worlds are points in time ($\Diamond \varphi$ means “$\varphi$ will be true”), assume actualist reading for now:

1. $\forall x \Diamond T(x)$
2. $\Diamond \forall x T(x)$
3. But do we mean, “tall” as we currently use the word tall, or as the word is used in the future?
(x: type 0, P: type ↑⟨0⟩, X: type ↑⟨0⟩)

⟨λX.◊(∃x)X(x)⟩(P) ↔ ◊⟨λX.(∃x)X(x)⟩(P) is valid

ℳ, Γ |= ν ⟨λX.◊(∃x)X(x)⟩(P)
(x: type 0, P: type ↑⟨0⟩, X: type ↑⟨0⟩)

$\langle \lambda X. \Diamond (\exists x)X(x) \rangle (P) \leftrightarrow \Diamond \langle \lambda X.(\exists x)X(x) \rangle (P)$ is valid

$\mathcal{M}, \Gamma \models_{v} \langle \lambda X. \Diamond (\exists x)X(x) \rangle (P)$

if $\mathcal{M}, \Gamma \models_{v} \Diamond (\exists x)X(x)[X/O]$ (where $O = \mathcal{I}(P, \Gamma)$)
(x: type 0, P: type ↑⟨0⟩, X: type ↑⟨0⟩)

⟨λX.◊(∃x)X(x)⟩(P) ↔ ◊⟨λX.(∃x)X(x)⟩(P) is valid

\[M, \Gamma \models_v  ⟨λX.◊(∃x)X(x)⟩(P)\]
iiff \[M, \Gamma \models_v ◊(∃x)X(x)[X/O]\] (where \(O = I(P, \Gamma)\))

iff there is a \(\Delta\) with \(\Gamma R \Delta\) and \(M, \Delta \models_v (∃x)P(x)\) (usual definition, \(P\) constant symbol)
(x: type 0, P: type ↑⟨0⟩, X: type ↑⟨0⟩)
⟨λX.◊(∃x)X(x)⟩(P) ↔ ◊⟨λX.(∃x)X(x)⟩(P) is valid

\(\mathcal{M}, \Gamma \models_P ⟨λX.◊(∃x)X(x)⟩(P)\)
iff \(\mathcal{M}, \Gamma \models_P ◊(∃x)X(x)[X/O]\) (where \(O = \mathcal{I}(P, \Gamma)\))

iff there is a \(\Delta\) with \(\Gamma R \Delta\) and \(\mathcal{M}, \Delta \models_P (∃x)P(x)\) (usual definition, \(P\) constant symbol)

iff \(\Gamma R \Delta\) and there is a \(a \in D\) such that \(a \in \mathcal{I}(P)(\Delta)\)
(x: type 0, P: type ↑⟨0⟩, X: type ↑⟨0⟩)

⟨λX.◊(∃x)X(x)⟩(P) ↔ ◊⟨λX.(∃x)X(x)⟩(P) is valid

\[ \mathcal{M}, \Gamma \models_v \langle \lambda X.\diamond(\exists x)X(x)\rangle(P) \]

iff \[ \mathcal{M}, \Gamma \models_v \diamond(\exists x)X(x)[X/O] \] (where \( O = \mathcal{I}(P, \Gamma) \))

iff there is a \( \Delta \) with \( \Gamma R \Delta \) and \( \mathcal{M}, \Delta \models_v (\exists x)P(x) \) (usual definition, \( P \) constant symbol)

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iff \( \Gamma R \Delta \) and \( \mathcal{M}, \Delta \models_v \exists xP(x) \)
\( (x: \text{ type } 0, P: \text{ type } \uparrow\langle 0 \rangle, X: \text{ type } \uparrow\langle 0 \rangle) \)

\[ \langle \lambda X. \Diamond (\exists x)X(x) \rangle (P) \leftrightarrow \Diamond \langle \lambda X.(\exists x)X(x) \rangle (P) \text{ is valid} \]

\( \mathcal{M}, \Gamma \models_v \langle \lambda X.\Diamond (\exists x)X(x) \rangle (P) \)

iff \( \mathcal{M}, \Gamma \models_v \Diamond (\exists x)X(x)[X/O] \) (where \( O = I(P, \Gamma) \))

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(x: type 0, \( P \): type \( \uparrow \langle 0 \rangle \), \( X \): type \( \uparrow \langle 0 \rangle \))

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iff \( \mathcal{M}, \Gamma \models_v \Diamond \langle \lambda X.(\exists x)X(x) \rangle (P) \)
(x: type 0, P: type \uparrow\langle0\rangle, X: type \uparrow\langle0\rangle)

\langle \lambda X.\Diamond (\exists x)X(x) \rangle(\downarrow P) \rightarrow \Diamond \langle \lambda X.(\exists x)X(x) \rangle(\downarrow P) \text{ is not valid}
\[(x: \text{ type } 0, P: \text{ type } \uparrow\langle 0 \rangle, X: \text{ type } \uparrow\langle 0 \rangle)\]

\[\langle \lambda X. \Diamond (\exists x)X(x) \rangle (\downarrow P) \rightarrow \Diamond \langle \lambda X. (\exists x)X(x) \rangle (\downarrow P) \text{ is not valid}\]

\[\Gamma \begin{array}{c} a \\ \hline \end{array} \quad I(P, \Gamma) = \{a\}\]

\[\Delta \begin{array}{c} a \\ \hline \end{array} \quad I(P, \Delta) = \emptyset\]

\[M, \Gamma \models_v \langle \lambda X. \Diamond (\exists x)X(x) \rangle (\downarrow P)\]
(x: type 0, P: type ↑⟨0⟩, X: type ↑⟨0⟩)

⟨λX.◊(∃x)X(x)⟩(↓P) → ◊⟨λX.(∃x)X(x)⟩(↓P) is not valid

Γ  ❍ a ✷ I(P, Γ) = {a}

Δ ❍ a ✷ I(P, Δ) = ∅

M, Γ ⊨v ⟨λX.◊(∃x)X(x)⟩(↓P)
\[(x: \text{ type } 0, \ P: \text{ type } \uparrow\langle 0 \rangle, \ X: \text{ type } \uparrow\langle 0 \rangle)\]

\[\langle \lambda X. \diamond(\exists x)X(x) \rangle(\downarrow P) \rightarrow \diamond \langle \lambda X. (\exists x)X(x) \rangle(\downarrow P) \text{ is not valid}\]

\[\Gamma \begin{array}{c}
\Box \ a \\
\hline
\end{array} \quad \mathcal{I}(P, \Gamma) = \{a\}\]

\[\Delta \begin{array}{c}
\Box \ a \\
\hline
\end{array} \quad \mathcal{I}(P, \Delta) = \emptyset\]

\[\mathcal{M}, \Gamma \models_{v} \langle \lambda X. \diamond(\exists x)X(x) \rangle(\downarrow P)\]

iff \[\mathcal{M}, \Gamma \models_{v} \diamond \exists xX(x)[X/\{a\}]\]
(x: type 0, P: type \uparrow\langle0\rangle, X: type \uparrow\langle0\rangle)

\langle\lambda X.\Diamond(\exists x)X(x)\rangle(\downarrow P) \rightarrow \Diamond\langle\lambda X.(\exists x)X(x)\rangle(\downarrow P) is not valid

\[ \Gamma \overset{a}{\rightarrow} ~ \mathcal{I}(P, \Gamma) = \{a\} \]

\[ \Delta \overset{a}{\rightarrow} ~ \mathcal{I}(P, \Delta) = \emptyset \]

\[ \mathcal{M}, \Gamma \models \Diamond\langle\lambda X.(\exists x)X(x)\rangle(\downarrow P) \]
(x: type \(0\), \(P\): type \(\uparrow\{0\}\), \(X\): type \(\uparrow\{0\}\))

\[\langle \lambda X.\Diamond(\exists x)X(x) \rangle(\downarrow P) \rightarrow \Diamond\langle \lambda X.\exists x)X(x) \rangle(\downarrow P) \text{ is not valid}\]

\[\Gamma \vdash a \quad \mathcal{I}(P, \Gamma) = \{a\}\]

\[\Delta \vdash a \quad \mathcal{I}(P, \Delta) = \emptyset\]

\[\mathcal{M}, \Gamma \models_{v} \Diamond\langle \lambda X.(\exists x)X(x) \rangle(\downarrow P)\]

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iff \(\Gamma \triangleright \Delta\) and \(\mathcal{M}, \Delta \models_{v} \exists xX(x)[X/\emptyset]\)
\[(x: \text{type } 0, P: \text{type } \uparrow\langle 0 \rangle, X: \text{type } \uparrow\langle 0 \rangle)\]

\[\langle \lambda X. \diamond (\exists x) X(x) \rangle (\downarrow P) \rightarrow \diamond \langle \lambda X. (\exists x) X(x) \rangle (\downarrow P) \text{ is not valid}\]

\[\Gamma \quad \text{□} \quad I(P, \Gamma) = \{a\}\]

\[\Delta \quad \text{□} \quad I(P, \Delta) = \emptyset\]

\[M, \Gamma \not\vdash_v \diamond \langle \lambda X. (\exists x) X(x) \rangle (\downarrow P)\]

iff \(\Gamma R \Delta \text{ and } M, \Delta \not\vdash_v \exists x X(x) (\downarrow P)\)

iff \(\Gamma R \Delta \text{ and } M, \Delta \not\vdash_v \exists x X(x)[X/\emptyset]\)
Tableaus
Possibly God exists

**Informal Axiom 1:** Exactly one of a property or its complement is positive
Possibly God exists

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Definition: $P$ entails $Q$ if, necessarily, everything having $P$ also has $Q$. 

Possibly God exists

**Informal Axiom 1:** Exactly one of a property or its complement is positive

**Definition:** $P$ entails $Q$ if, necessarily, everything having $P$ also has $Q$.

**Informal Axiom 2:** Any property entailed by a positive property is positive
Possibly God exists

**Informal Axiom 1:** Exactly one of a property or its complement is positive.

**Definition:** $P$ entails $Q$ if, necessarily, everything having $P$ also has $Q$.

**Informal Axiom 2:** Any property entailed by a positive property is positive.

**Informal Proposition 1:** Any positive property is possibly instantiated. I.e., if $P$ is positive then it is possible that something has property $P$. 
Possibly God exists

**Informal Axiom 3:** The conjunction of any collection of positive properties is positive.
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**Informal Definition:** A God is any being that has every positive property.
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**Informal Proposition 2:** It is possible that God exists.
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**Informal Axiom 3:** The conjunction of any collection of positive properties is positive.

**Informal Definition:** A God is any being that has every positive property

**Informal Proposition 2:** It is possible that God exists.
God’s existence is necessary, if possible

**Definition** A property $G$ is the **essence** of an object $g$ if:

1. $g$ has property $G$
2. $G$ entails every property of $g$
God’s existence is necessary, if possible

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1. $g$ has property $G$
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**Informal Proposition:** If $g$ is a God, the essence of $g$ is being a God.
God’s existence is necessary, if possible

**Definition** An object \( g \) has the property of **necessary existing** if the essence of \( g \) is necessarily instantiated.
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**Informal Axiom 5:** Necessary existence, itself, is a positive property.
God’s existence is necessary, if possible

**Definition** An object $g$ has the property of **necessary existing** if the essence of $g$ is necessarily instantiated.

**Informal Axiom 5**: Necessary existence, itself, is a positive property.

**Informal Proposition** If a God exists, a God exists necessarily.
God’s existence is necessary, if possible

**Definition** An object \( g \) has the property of **necessary existing** if the essence of \( g \) is necessarily instantiated.

**Informal Axiom 5**: Necessary existence, itself, is a positive property.

**Informal Proposition** If a God exists, a God exists necessarily.

**Informal Proposition** If it is possible that a God exists, it is necessary that a God exists (assume S5)
**Informal Theorem** Assuming all the axioms, and assuming that the underlying logic is **S5**, a (the) God necessarily exists.
Formalizing Proposition 1

**Definition:** Let \( P \) represent *positiveness*. \( P \) is a constant symbol of type \( \uparrow \langle \uparrow 0 \rangle \). \( P \) is positive if we have \( P(P) \).

**Definition** If \( \tau \) is a term of type \( \uparrow \langle 0 \rangle \), take \( \neg \tau \) as short for \( \langle \lambda x. \neg \tau(x) \rangle \). Call \( \tau \) negative if \( \neg \tau \) is positive.
Formalizing Proposition 1

**Definition:** Let $\mathcal{P}$ represent **positiveness**. $\mathcal{P}$ is a constant symbol of type $\uparrow\langle\uparrow 0\rangle$. $\mathcal{P}$ is positive if we have $\mathcal{P}(\mathcal{P})$.

**Definition** If $\tau$ is a term of type $\uparrow\langle 0\rangle$, take $\neg\tau$ as short for $\langle \lambda x. \neg\tau(x) \rangle$. Call $\tau$ negative if $\neg\tau$ is positive.

**Formalizing Axiom 1** (Axiom 11.3)

1. $\forall X[\mathcal{P}(\neg X) \rightarrow \neg\mathcal{P}(X)]$
2. $\forall X[\neg\mathcal{P}(X) \rightarrow P(X)]$
Formalizing Proposition 1

Formalizing Axiom 2 (Axiom 11.5)

\[(\forall X)(\forall Y)[\mathcal{P}(X) \land \Box(\forall^E x)(X(x) \rightarrow Y(x))] \rightarrow \mathcal{P}(Y)]\]
Formalizing Proposition 1

Formalizing Axiom 2 (Axiom 11.5)

$$\forall X \forall Y [P(X) \land \Box (\forall^E x) (X(x) \rightarrow Y(x))] \rightarrow P(Y)$$

Proposition Assuming 11.5
1. $$\exists X P(X) \rightarrow P(\langle \lambda x. x = x \rangle)$$
2. $$\exists X P(X) \rightarrow P(\neg \langle x. \neg x = x \rangle)$$
Formalizing Proposition 1

Formalizing Axiom 2 (Axiom 11.5)

$$(\forall X)(\forall Y)[[P(X) \land \Box(\forall^E x)(X(x) \rightarrow Y(x))]] \rightarrow P(Y)]$$

Proposition Assuming 11.5
1. $$(\exists X)P(X) \rightarrow P(\langle \lambda x. x = x \rangle)$$
2. $$(\exists X)P(X) \rightarrow P(\neg \langle x. \neg x = x \rangle)$$

Proposition Assuming 11.3 A and 11.5
$$(\exists X)P(X) \rightarrow \neg P(\langle \lambda x. \neg x = x \rangle)$$
Formalizing Proposition 1

**Formalizing Axiom 2** (Axiom 11.5)

\[(\forall X)(\forall Y)[[P(X) \land \Box(\forall^E x)(X(x) \rightarrow Y(x))] \rightarrow P(Y)]\]

**Proposition** Assuming 11.5

1. \((\exists X)P(X) \rightarrow P(\langle \lambda x.x = x \rangle)\)
2. \((\exists X)P(X) \rightarrow P(\neg\langle x.\neg x = x \rangle)\)

**Proposition** Assuming 11.3 A and 11.5

\((\exists X)P(X) \rightarrow \neg P(\langle \lambda x.\neg x = x \rangle)\)

**Formalizing Informal Proposition 1** Assuming 11.3 A and 11.5

\[(\forall X)[P(X) \rightarrow \Diamond(\exists^E x)X(x)]\]
Formalizing Informal Axiom 3

**Axiom 11.9:** \((\forall X)(\forall Y)[\mathcal{P}(X) \land \mathcal{P}(Y)] \rightarrow \mathcal{P}(X \land Y)]\)
Formalizing Informal Axiom 3

**Axiom 11.9:** \((\forall X)(\forall Y)[[P(X) \land P(Y)] \rightarrow P(X \land Y)]\)

*But this should hold for any number of Xs*
Axiom 11.9: \((\forall X)(\forall Y)[[P(X) \land P(Y)] \rightarrow P(X \land Y)]\)

But this should hold for any number of \(X\)s

1. \(\mathcal{Z}\) applies to only positive properties:

\[
pos(\mathcal{Z}) := (\forall X)[\mathcal{Z}(X) \rightarrow P(X)]
\]

2. \(X\) is the (necessary) intersection of \(\mathcal{Z}\)

\[
(X \text{ intersection of } \mathcal{Z}) := \Box(\forall x)[X(x) \leftrightarrow (\forall Y)[\mathcal{Z}(Y) \rightarrow Y(x)]]
\]
Formalizing Informal Axiom 3

Axiom 11.9: \((\forall X)(\forall Y)[[P(X) \land P(Y)] \rightarrow P(X \land Y)]\)
But this should hold for any number of Xs

1. \(Z\) applies to only positive properties:

\[
p \text{pos}(Z) := (\forall X)[Z(X) \rightarrow P(X)]
\]

2. \(X\) is the (necessary) intersection of \(Z\)

\[
(X \text{ intersection of } Z) := \square(\forall x)[X(x) \leftrightarrow (\forall Y)[Z(Y) \rightarrow Y(x)]]
\]

Axiom 11.10:

\((\forall Z)[\text{pos}(Z) \rightarrow \forall X[(X \text{ intersection of } Z) \rightarrow P(X)]]\)
Technical Assumptions (Axiom 4)

\[(\forall X)[\mathcal{P}(X) \rightarrow \Box \mathcal{P}(X)]\]

\[(\forall X)[\neg \mathcal{P}(X) \rightarrow \Box \neg \mathcal{P}(X)]\]
Technical Assumptions (Axiom 4)

\[(\forall X)[P(X) \rightarrow \Box P(X)]\]

\[(\forall X)[\neg P(X) \rightarrow \Box \neg P(X)]\]

“because it follows from he nature of the property” -Gödel.
Technical Assumptions (Axiom 4)

$(\forall X)[P(X) \rightarrow \Box P(X)]$

$(\forall X)[\neg P(X) \rightarrow \Box \neg P(X)]$

"because it follows from he nature of the property" - Gödel.

Axiom 11.11: $(\forall X)[P(X) \rightarrow \Box P(X)].$
Being Godlike

**Godlike** is an intension term of type $\uparrow\langle 0 \rangle$, intuitively the set of god-like objects at a world.

**Definition 11.12** $G$ is the following type $\uparrow\langle 0 \rangle$ term:

$$\langle \lambda x. (\forall Y)[P(Y) \rightarrow Y(x)] \rangle$$

**Definition 11.13** $G^*$ is the following type $\uparrow\langle 0 \rangle$ term:

$$\langle \lambda x. (\forall Y)[P(Y) \leftrightarrow Y(x)] \rangle$$
Being Godlike

**Godlike** is an intension term of type $\uparrow\langle0\rangle$, intuitively the set of god-like objects at a world.

**Definition 11.12** $G$ is the following type $\uparrow\langle0\rangle$ term:

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**Definition 11.13** $G^*$ is the following type $\uparrow\langle0\rangle$ term:

$$\langle \lambda x. (\forall Y)[P(Y) \leftrightarrow Y(x)] \rangle$$

**Proposition** Assuming 11.3B, in $K$, $(\forall x)[G(x) \leftrightarrow G^*(x)]$. 
Possibly God exists

**Theorem 11.17** Assume axioms 11.3A, 11.5 and 11.10. In $K$ both of the following are consequences: $\Diamond(\exists^E x)G(x)$ and $\Diamond(\exists x)G(x)$. 
Objection 1

**Theorem** Assume all the axioms except for 11.10 and 11.9, the following are equivalent using $S5$:

1. Axiom 11.10:
   \[(\forall Z)[pos(Z) \rightarrow \forall X[(X \text{ intersection of } Z) \rightarrow P(X)]]\]
2. $P(G)$
3. $\Diamond (\exists^E x) G(x)$
Necessarily God exists

**Formalizing Informal Definition 6** Let $N$ abbreviate the following type $\uparrow\langle 0 \rangle$ term:

$$\langle \lambda x. (\forall Y) [E(Y, x) \rightarrow \Box (\exists^E z Y(z))] \rangle$$

something has property $N$ of necessary existence provided any essence of it is necessarily instantiated.
Necessarily God exists

**Formalizing Informal Definition 6** Let $N$ abbreviate the following type $\uparrow\langle 0 \rangle$ term:

$$\langle \lambda x. (\forall Y)[E(Y,x) \rightarrow \Box (\exists zY(z))] \rangle$$

Something has property $N$ of necessary existence provided any essence of it is necessarily instantiated.

**Axiom 11.25**: $\mathcal{P}(N)$.
Essence

The essence of something, \( x \), is a property that entails every property that \( x \) possesses: Intuitively,

\[
(\varphi \text{ Ess } x) \leftrightarrow \varphi(x) \land (\forall \psi)[\psi(x) \rightarrow \square \forall y[\varphi(y) \rightarrow \psi(y)]]
\]

Definition \( \mathcal{E} \) abbreviates the following \( \uparrow\langle\uparrow\langle 0 \rangle, 0 \rangle \), term (\( Z \) is type \( \uparrow\langle 0 \rangle \) and \( w \) is type 0):

\[
\langle \lambda Y, x. Y(x) \land \forall Z[Z(x) \rightarrow \square (\forall^E w)[Y(w) \rightarrow Z(w)]]\rangle
\]
Essence

The **essence** of something, \( x \), is a property that **entails** every property that \( x \) possesses: Intuitively,

\[
(\varphi \text{ Ess } x) \iff \varphi(x) \land (\forall \psi)[\psi(x) \rightarrow \Box \forall y[\varphi(y) \rightarrow \psi(y)]
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**Definition** \( \mathcal{E} \) abbreviates the following \( \uparrow\langle\uparrow\langle0\rangle, 0\rangle \), term (\( Z \) is type \( \uparrow\langle0\rangle \) and \( w \) is type 0):

\[
\langle \lambda Y, x. Y(x) \land \forall Z[Z(x) \rightarrow \Box (\forall^E w)[Y(w) \rightarrow Z(w)]] \rangle
\]

**Theorem** Assume axioms 11.3B and 11.11, in \( K \) the following is provable: \( (\forall x)[G(x) \rightarrow \mathcal{E}(G, x)] \) (same for \( G^* \)).
Essence

The **essence** of something, $x$, is a property that *entails* every property that $x$ possesses: Intuitively,

$$(\varphi \text{ Ess } x) \leftrightarrow \varphi(x) \land (\forall \psi)[\psi(x) \rightarrow \Box \forall y[\varphi(y) \rightarrow \psi(y)]]$$

**Definition** $E$ abbreviates the following $\uparrow\langle\uparrow\langle 0 \rangle, 0 \rangle$, term ($Z$ is type $\uparrow\langle 0 \rangle$ and $w$ is type 0):

$$\langle \lambda Y, x. Y(x) \land \forall Z[Z(x) \rightarrow \Box(\forall E w)[Y(w) \rightarrow Z(w)]] \rangle$$

**Theorem** Assume axioms 11.3B and 11.11, in $K$ the following is provable: $(\forall x)[G(x) \rightarrow E(G, x)]$ (same for $G^*$).

**Theorem** In $K$, the following is provable

$$(\forall X)(\forall y)[E(X, y) \rightarrow \Box(\forall E z[X(z) \rightarrow (y = z)]]$$
Necessarily God exists

Theorem Assume Axioms 11.3B, 11.11, 11.25, in \( \mathbf{K} \)

\[(\exists x)G(x) \rightarrow \Box(\exists^E x)G(x)\]
Necessarily God exists

**Theorem** Assume Axioms 11.3B, 11.11, 11.25, in $\mathbf{K}$

$$(\exists x)G(x) \rightarrow \square(\exists^E x)G(x)$$

**Theorem** Assume axioms 11.3B, 11.11, 11.25, In the logic $\mathbf{S5}$,

$$\Diamond(\exists x)G(x) \rightarrow \square(\exists^E x)G(x)$$
Necessarily God exists

**Theorem** Assume Axioms 11.3B, 11.11, 11.25, in $K$

$$(\exists x)G(x) \rightarrow \Box(\exists^Ex)G(x)$$

**Theorem** Assume axioms 11.3B, 11.11, 11.25, In the logic $S5$,

$$\Diamond(\exists x)G(x) \rightarrow \Box(\exists^Ex)G(x)$$

**Corollary** $\Box(\exists^Ex)G(x)$
Conclusions

- Other objections: the modal system collapses ($Q \rightarrow \Box Q$ is valid)
- Fitting has a number of papers which develops and applies (fragments of) this framework (papers on Database Theory, logics “between” propositional and first order.
- ....