Logics of Rational Agency

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We are interested in reasoning about rational agents interacting in *social* situations.
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- Philosophy (social philosophy, epistemology)
- Game Theory
- Social Choice Theory
- AI (multiagent systems)
We are interested in reasoning about rational agents interacting in social situations.

**What is a rational agent?**

- maximize expected utility (instrumentally rational)
- react to observations
- revise beliefs when learning a *surprising* piece of information
- understand higher-order information
- plans for the future
- asks questions
- ????
We are interested in reasoning about rational agents interacting in social situations.

There is a jungle of formal systems!

- logics of informational attitudes (knowledge, beliefs, certainty)
- logics of action & agency
- temporal logics/dynamic logics
- logics of motivational attitudes (preferences, intentions)

(Not to mention various game-theoretic/social choice models and logical languages for reasoning about them)
Introduction and Motivation

We are interested in reasoning about rational agents interacting in social situations.

There is a jungle of formal systems!

- How do we compare different logical systems studying the same phenomena?
- How complex is it to reason about rational agents?
- (How) should we merge the various logical systems?
- What do the logical frameworks contribute to the discussion on rational agency?

and logical languages for reasoning about them)
We are interested in reasoning about rational agents interacting in social situations.

- playing a game (e.g., a card game)
- having a conversation
- executing a social procedure
- ....
What about game-theoretic analyses?

Goal: incorporate/extend existing game-theoretic/social choice analyses
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Introduction and Motivation

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Formally, a game is described by its strategy sets and payoff functions. But in real life, many other parameters are relevant; there is a lot more going on. Situations that substantively are vastly different may nevertheless correspond to precisely the same strategic game. For example, in a parliamentary democracy with three parties, the winning coalitions are the same whether the parties hold a third of the seats, or, say, 49%, 39%, and 12% respectively. But the political situations are quite different. The difference lies in the attitudes of the players, in their expectations about each other, in custom, and in history, though the rules of the game do not distinguish between the two situations.

Logics of Rational Agency
Basic Ingredients

- What are the basic building blocks? (the nature of time (continuous or discrete/branching or linear), how (primitive) events or actions are represented, how causal relationships are represented and what constitutes a state of affairs.)
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▶ Single agent vs. many agents.
Basic Ingredients

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- Single agent vs. many agents.

- What are the primitive operators?
  - Informational attitudes
  - Motivational attitudes
  - Normative attitudes
Basic Ingredients

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- Single agent vs. many agents.

- What are the primitive operators?
  - Informational attitudes
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- Static vs. dynamic
Basic Ingredients

- informational attitudes
- time, actions and ability
- motivational attitudes
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Single-Agent Epistemic Logic

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- $LP$: “$P$ is an epistemic possibility”
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LP: & \text{ “} P \text{ is an epistemic possibility”} \\
KLP: & \text{ “Ann knows that she thinks } P \text{ is possible”}
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Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.
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What are the relevant states?
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Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

Ann receives card 3 and card 1 is put on the table
Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

What information does Ann have?
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Suppose $H_i$ is intended to mean “Ann has card $i$”

$T_i$ is intended to mean “card $i$ is on the table”

Eg., $V(H_1) = \{w_1, w_2\}$
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$\mathcal{M}, w_1 \models KH_1$
Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

\[ M, w_1 \models K H_1 \]
Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

\[ M, w_1 \models K H_1 \]

\[ M, w_1 \models K \neg T_1 \]
Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

\[ M, w_1 \models LT_2 \]
Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

\[ \mathcal{M}, w_1 \models K(T_2 \lor T_3) \]
Basic Ingredients

The Language

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \land \psi \mid K \varphi \]

Kripke Models: \( \mathcal{M} = \langle W, R, V \rangle \) and \( w \in W \)

Truth: \( \mathcal{M}, w \models \varphi \) is defined as follows:

- \( \mathcal{M}, w \models p \) iff \( w \in V(p) \) (with \( p \in \text{At} \))
- \( \mathcal{M}, w \models \neg \varphi \) if \( \mathcal{M}, w \not\models \varphi \)
- \( \mathcal{M}, w \models \varphi \land \psi \) if \( \mathcal{M}, w \models \varphi \) and \( \mathcal{M}, w \models \psi \)
- \( \mathcal{M}, w \models K \varphi \) if for each \( v \in W \), if \( wRv \), then \( \mathcal{M}, v \models \varphi \)
Basic Ingredients

Some Questions

Should we make additional assumptions about $R$ (i.e., reflexive, transitive, etc.)

What idealizations have we made?
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<thead>
<tr>
<th>Modal Formula</th>
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The Language: $\varphi := p \mid \neg \varphi \mid \varphi \land \psi \mid K_i \varphi$ with $i \in \mathcal{A}$

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Basic Ingredients

Multi-agent Epistemic Logic

- $K_A K_B \varphi$: “Ann knows that Bob knows $\varphi$”

- $K_A (K_B \varphi \lor K_B \neg \varphi)$: “Ann knows that Bob knows whether $\varphi$”

- $\neg K_B K_A K_B (\varphi)$: “Bob does not know that Ann knows that Bob knows that $\varphi$”
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Suppose that Ann receives card 1 and card 2 is on the table.
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\[ \mathcal{M}, w \models K_B(K_A H_1 \lor K_A \neg H_1) \]
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**Group Knowledge**

$K_A P$: “Ann knows that $P$”
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$K_A K_B P$: “Ann knows that Bob knows that $P$”

$K_A P \land K_B P$: “Every one knows $P$.”
Group Knowledge

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$K_A P \land K_B P$: “Every one knows $P$”. Let $EP := K_A P \land K_B P$
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\( K_A EP \): “Ann knows that everyone knows that \( P \)” .
Group Knowledge

*KAP: “Ann knows that \( P \)”

*KBP: “Bob knows that \( P \)”

*KAKBP: “Ann knows that Bob knows that \( P \)”

*KAP \& KBP: “Every one knows \( P \)” let \( EP := KAP \& KBP \)

*KAPEP: “Ann knows that everyone knows that \( P \)”.

*EEP: “Everyone knows that everyone knows that \( P \)”.
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\(K_A P\): “Ann knows that \(P\)”

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\(K_A P \land K_B P\): “Every one knows \(P\)” . Let \(EP \coloneqq K_A P \land K_B P\)

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\(EEEP\): “Everyone knows that everyone knows that everyone knows that \(P\)” .
Basic Ingredients

Common Knowledge

CP: “It is common knowledge that $P$”
Common Knowledge

*CP:* “It is common knowledge that $P$” — “Everyone knows that everyone knows that everyone knows that $\cdots$ $P$”.
Common Knowledge

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Is common knowledge different from everyone knows?
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$w_1 \models EP \land \neg CP$
Basic Ingredients

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Is *common knowledge* different from *everyone knows*?

\[
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\quad & \text{A} \\
\quad & \text{w}_2 \\
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\quad & \text{B} \\
\quad & \text{A, B} \\
\quad & \text{P} \\
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\end{align*}
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Common Knowledge

*CP*: “It is **common knowledge** that *P***” — “Everyone knows that everyone knows that everyone knows that ... *P***.

Is **common knowledge** different from **everyone knows**?

\[ w_1 \models EP \land \neg CP \]
The operator “everyone knows \( P \)”, denoted \( EP \), is defined as follows

\[ EP := \bigwedge_{i \in \mathcal{A}} K_i P \]

\( w \models CP \) iff every finite path starting at \( w \) ends with a state satisfying \( P \).
Suppose you are told "Ann and Bob are going together," and respond "sure, that's common knowledge." What you mean is not only that everyone knows this, but also that the announcement is pointless, occasions no surprise, reveals nothing new; in effect, that the situation after the announcement does not differ from that before.

...the event "Ann and Bob are going together" — call it $P$ — is common knowledge if and only if some event — call it $Q$ — happened that entails $P$ and also entails all players' knowing $Q$ (like all players met Ann and Bob at an intimate party).

(Robert Aumann)
CP → ECP

Suppose you are told “Ann and Bob are going together,” and respond “sure, that’s common knowledge.” What you mean is not only that everyone knows this, but also that the announcement is pointless, occasions no surprise, reveals nothing new; in effect, that the situation after the announcement does not differ from that before. ...the event “Ann and Bob are going together” — call it $P$ — is common knowledge if and only if some event — call it $Q$ — happened that entails $P$ and also entails all players’ knowing $Q$ (like all players met Ann and Bob at an intimate party). (Robert Aumann)
Basic Ingredients

\[ P \land C(P \to EP) \to CP \]
An Example

Two players Ann and Bob are told that the following will happen. Some positive integer $n$ will be chosen and one of $n$, $n + 1$ will be written on Ann’s forehead, the other on Bob’s. Each will be able to see the other’s forehead, but not his/her own.
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Suppose the number are (2,3).
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Do the agents know there numbers are less than 1000?
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Suppose the number are (2,3).

Do the agents know there numbers are less than 1000?

Is it common knowledge that their numbers are less than 1000?
Basic Ingredients

✓ informational attitudes (knowledge, group knowledge, belief, certainty, etc.)

▶ time, actions and ability

▶ motivational attitudes
Actions: Two Views
Basic Ingredients

Actions: Two Views

1. Actions \textit{transition between states, or situations}

\begin{tikzpicture}
  \node[draw, circle] (s) at (0,0) {$s$};
  \node[draw, circle] (t) at (1,0) {$t$};
  \draw[->] (s) to node [above] {$a$} (t);
\end{tikzpicture}
Basic Ingredients

Actions: Two Views

1. Actions *transition between states, or situations*

2. Actions restrict the set of *possible future histories*
Propositional Dynamic Logic

Semantics: \( \mathcal{M} = \langle W, \{ R_a \mid a \in P \}, V \rangle \) where for each \( a \in P \), \( R_a \subseteq W \times W \) and \( V : \text{At} \rightarrow \wp(W) \)

- \( R_{\alpha \cup \beta} := R_\alpha \cup R_\beta \)
- \( R_{\alpha ; \beta} := R_\alpha \circ R_\beta \)
- \( R_\alpha^* := \bigcup_{n \geq 0} R_\alpha^n \)
- \( R_\varphi ? = \{(w, w) \mid \mathcal{M}, w \models \varphi \} \)

\( \mathcal{M}, w \models [\alpha] \varphi \) iff for each \( v \), if \( wR_\alpha v \) then \( \mathcal{M}, v \models \varphi \)
Basic Ingredients

Background: Propositional Dynamic Logic

1. Axioms of propositional logic
2. 
   \[ [\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\varphi \rightarrow [\alpha]\psi) \]
3. \[ [\alpha \cup \beta]\varphi \leftrightarrow [\alpha]\varphi \land [\beta]\varphi \]
4. \[ [\alpha; \beta]\varphi \leftrightarrow [\alpha][\beta]\varphi \]
5. \[ [\psi?]\varphi \leftrightarrow (\psi \rightarrow \varphi) \]
6. \[ \varphi \land [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi \]
7. \[ \varphi \land [\alpha^*](\varphi \rightarrow [\alpha]\varphi) \rightarrow [\alpha^*]\varphi \]
8. Modus Ponens and Necessitation (for each program \( \alpha \))
Background: Propositional Dynamic Logic

1. Axioms of propositional logic

2. \([\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\varphi \rightarrow [\alpha]\psi)\)

3. \([\alpha \cup \beta]\varphi \leftrightarrow [\alpha]\varphi \land [\beta]\varphi\)

4. \([\alpha; \beta]\varphi \leftrightarrow [\alpha][\beta]\varphi\)

5. \([\psi?]\varphi \leftrightarrow (\psi \rightarrow \varphi)\)

6. \(\varphi \land [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi\) (Fixed-Point Axiom)

7. \(\varphi \land [\alpha^*](\varphi \rightarrow [\alpha]\varphi) \rightarrow [\alpha^*]\varphi\) (Induction Axiom)

8. Modus Ponens and Necessitation (for each program \(\alpha\))
Propositional Dynamic Logic

**Theorem PDL** is sound and weakly complete with respect to the Segerberg Axioms.

**Theorem** The satisfiability problem for PDL is decidable (EXPTIME-Complete).


An early approach to interpret PDL as logic of actions was put forward by Krister Segerberg.

Segerberg adds an “agency” program to the PDL language $\delta A$ where $A$ is a formula.

Actions and Agency

The intended meaning of the program ‘\(\delta A\)’ is that the agent “brings it about that \(A\)”: formally, \(\delta A\) is the set of all paths \(p\) such that
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Interestingly, Segerberg also briefly considers a third condition:

3. \(p\) is optimal (in some sense: shortest, maximally efficient, most convenient, etc.) in the set of computations satisfying conditions (1) and (2).
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The axioms:

1. [δA]A
2. [δA]B → ([δB]C → [δA]C)
Actions and Agency

Logics of Action and Agency

Alternative accounts of agency do not include explicit description of the actions:
Each node represents a choice point for the agent.

A **history** is a maximal branch in the above tree.

Formulas are interpreted at history moment pairs.

At each moment there is a choice available to the agent (partition of the histories through that moment)

The key modality is \([\text{stit}]\varphi\) which is intended to mean that the agent \(i\) can “see to it that \(\varphi\) is true”.

- \([\text{stit}]\varphi\) is true at a history moment pair provided the agent can choose a (set of) branch(es) such that every future history-moment pair satisfies \(\varphi\)
We use the modality ‘◊’ to mean historic possibility.

◊[stit]φ: “the agent has the ability to bring about φ”.
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Example Consider the example of an agent (call her Ann) throwing a dart. Suppose Ann is not a very good dart player, but she just happens to throw a bull’s eye.
We use the modality ‘◊’ to mean historic possibility.

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**Example** Consider the example of an agent (call her Ann) throwing a dart. Suppose Ann is not a very good dart player, but she just happens to throw a bull’s eye. Intuitively, we do not want to say that Ann has the *ability* to throw a bull’s eye even though it happens to be true. That is, the following principle should be falsifiable:

φ → ◊[STIT]φ
**Example** Continuing with this example, suppose that Ann has the ability to hit the dart board, but has no other control over the placement of the dart. Now, when she throws the dart, as a matter of fact, it will either hit the top half of the board or the bottom half of the board. Since, Ann has the ability to hit the dart board, she has the ability to either hit the top half of the board or the bottom half of the board.
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\[ \Diamond [stit] (\varphi \lor \psi) \rightarrow \Diamond [stit] \varphi \lor \Diamond [stit] \psi \]
The following model will falsify both of the above formulas:
Temporal Logics
Computational vs. Behavioral Structures
Computational vs. Behavioral Structures

\[ x = 1 \]

\[ x = 2 \]
Computational vs. Behavioral Structures

\[ x = 1 \]
\[ x = 2 \]
Temporal Logics

Linear Time Temporal Logic: Reasoning about computation paths:

\( \phi \): \( \phi \) is true some time in the future.


Branching Time Temporal Logic: Allows quantification over paths:

\( \exists \diamond \phi \): there is a path in which \( \phi \) is eventually true.

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  \[ \exists \Diamond \varphi: \text{there is a path in which } \varphi \text{ is eventually true.} \]

Temporal Logics

$\exists \diamond P_{x=2}$
**Basic Ingredients**

**Temporal Logics**

$\forall x \leq 1 \quad q_0$

$\forall x = 2 \quad q_1$

\[ \neg \forall \Diamond P_{x=2} \]

\[ \vdots \]
Many Agents

The previous model assumes there is one agent that “controls” the transition system.
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What if there is more than one agent?
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What if there is more than one agent?

**Example:** Suppose that there are two agents: a server \((s)\) and a client \((c)\). The client asks to set the value of \(x\) and the server can either grant or deny the request. Assume the agents make simultaneous moves.
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<tbody>
<tr>
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<td></td>
<td>$q_0 \Rightarrow q_0$, $q_1 \Rightarrow q_0$</td>
</tr>
<tr>
<td><em>set2</em></td>
<td></td>
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From Temporal Logic to Strategy Logic

- **Coalitional Logic**: Reasoning about (local) group power.

  \[ [C]\varphi \text{: coalition } C \text{ has a } \textbf{joint action} \text{ to bring about } \varphi. \]

From Temporal Logic to Strategy Logic

- **Coalitional Logic**: Reasoning about (local) group power.

  \[ [C] \varphi: \text{coalition } C \text{ has a joint action to bring about } \varphi. \]


- **Alternating-time Temporal Logic**: Reasoning about (local and global) group power:

  \[ \langle\langle A\rangle\rangle \square \varphi: \text{The coalition } A \text{ has a joint action to ensure that } \varphi \text{ will remain true.} \]

Multi-agent Transition Systems

\[ (P_{x=1} \rightarrow [s]P_{x=1}) \land (P_{x=2} \rightarrow [s]P_{x=2}) \]
Basic Ingredients

Multi-agent Transition Systems

\[ P_{x=1} \rightarrow \neg [s] P_{x=2} \]

\[ \langle *, \text{deny} \rangle \]

\[ q_0 \quad x = 1 \]

\[ \langle \text{set2, grant} \rangle \]

\[ q_1 \quad x = 2 \]

\[ \langle \text{set1, grant} \rangle \]

\[ \langle *, \text{deny} \rangle \]
Multi-agent Transition Systems

\[
\begin{align*}
q_0 & \xrightarrow{x=1} \langle *, \text{deny} \rangle \\
q_1 & \xrightarrow{x=2} \langle \text{set}2, \text{grant} \rangle \\
q_0 & \xrightarrow{\langle \text{set}1, \text{grant} \rangle} q_1 \\
q_0 & \xrightarrow{\langle *, \text{deny} \rangle} q_0 \\
q_1 & \xrightarrow{\langle *, \text{deny} \rangle} q_1
\end{align*}
\]

\[P_{x=1} \rightarrow \neg [s, c] P_{x=2}\]
Basic Ingredients

✓ informational attitudes (knowledge, group knowledge, belief, certainty, etc.)

✓ time, actions and ability (individual and coalitional ability)

▶ motivational attitudes
Preference (Modal) Logics

\( x, y \) objects

\( x \succeq y: x \text{ is at least as good as } y \)
Preference (Modal) Logics

$x, y$ objects

$x \succeq y$: $x$ is at least as good as $y$

1. $x \succeq y$ and $y \not\succeq x$ (\(x \succ y\))
2. $x \not\succeq y$ and $y \succeq x$ (\(y \succ x\))
3. $x \succeq y$ and $y \succeq x$ (\(x \sim y\))
4. $x \not\succeq y$ and $y \not\succeq x$ (\(x \perp y\))
Basic Ingredients

Preference (Modal) Logics

$x, y$ objects

$x \succeq y$: $x$ is at least as good as $y$

1. $x \succeq y$ and $y \not\succeq x$ ($x \succ y$)
2. $x \not\succeq y$ and $y \succeq x$ ($y \succ x$)
3. $x \succeq y$ and $y \succeq x$ ($x \sim y$)
4. $x \not\succeq y$ and $y \not\succeq x$ ($x \perp y$)

Properties: transitivity, connectedness, etc.
Preference (Modal) Logics

Modal betterness model $\mathcal{M} = \langle W, \succeq, V \rangle$
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Preference Modalities $\langle \succeq \rangle \varphi$: “there is a world at least as good (as the current world) satisfying $\varphi$”

$\mathcal{M}, w \models \langle \succeq \rangle \varphi$ iff there is a $v \succeq w$ such that $\mathcal{M}, v \models \varphi$
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$\mathcal{M}, w \models \langle \succ \rangle \varphi$ iff there is $v \succeq w$ and $w \not\succeq v$ such that $\mathcal{M}, v \models \varphi$
Preference (Modal) Logics

1. $\langle \neg \rangle \varphi \rightarrow \langle \geq \rangle \varphi$
2. $\langle \geq \rangle \langle \neg \rangle \varphi \rightarrow \langle \neg \rangle \varphi$
3. $\varphi \land \langle \geq \rangle \psi \rightarrow (\langle \neg \rangle \psi \lor \langle \geq \rangle (\psi \land \langle \geq \rangle \varphi))$
4. $\langle \neg \rangle \langle \geq \rangle \varphi \rightarrow \langle \neg \rangle \varphi$

**Theorem** The above logic (with Necessitation and Modus Ponens) is sound and complete with respect to the class of preference models.

Preference Modalities

\( \varphi \geq \psi \): the state of affairs \( \varphi \) is at least as good as \( \psi \) (ceteris paribus)

Basic Ingredients

From worlds to sets and back

**Lifting**

- \( X \geq_{\forall \exists} Y \) if \( \forall y \in Y \ \exists x \in X: x \succeq y \)

From worlds to sets and back

Lifting

\[ X \geq_{\forall \exists} Y \text{ if } \forall y \in Y \ \exists x \in X: x \succeq y \]

\[ A(\varphi \rightarrow \langle \succeq \rangle \psi) \]
From worlds to sets and back

Lifting

\[ X \geq_{\forall \exists} Y \text{ if } \forall y \in Y \ \exists x \in X: x \succeq y \]

\[ A(\varphi \rightarrow \langle \succeq \rangle \psi) \]

\[ X \geq_{\forall \forall} Y \text{ if } \forall y \in Y \ \forall x \in X: x \succeq y \]

\[ A(\varphi \rightarrow [\sim \sim] \neg \psi) \]
From worlds to sets and back

Lifting

- $X \geq_\forall \exists Y$ if $\forall y \in Y \ \exists x \in X: x \succeq y$
  $A(\varphi \rightarrow \langle \succeq \rangle \psi)$

- $X \geq_\forall \forall Y$ if $\forall y \in Y \ \forall x \in X: x \succeq y$
  $A(\varphi \rightarrow [\succeq] \neg \psi)$

Deriving

$P_1 \gg P_2 \gg P_3 \gg \cdots \gg P_n$

$x > y$ iff $x$ and $y$ differ in at least one $P_i$ and the first $P_i$ where this happens is one with $P_i x$ and $\neg P_i y$

The Logic of Group Decisions
The Logic of Group Decisions

**Fundamental Problem:** groups are inconsistent!
The Logic of Group Decisions: The Doctrinal “Paradox” (Kornhauser and Sager 1993)

\[ p \land q \leftrightarrow r \]

- \( p \): a valid contract was in place
- \( q \): there was a breach of contract
- \( r \): the court is required to find the defendant liable.

<table>
<thead>
<tr>
<th></th>
<th>( p )</th>
<th>( q )</th>
<th>( (p \land q) \leftrightarrow r )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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The Logic of Group Decisions: The Doctrinal “Paradox”
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Should we accept \( r \)?

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Should we accept $r$? No, a simple majority votes no.

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Should we accept $r$? Yes, a majority votes yes for $p$ and $q$ and
$(p \land q) \leftrightarrow r$ is a legal doctrine.

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Discursive Dilemma

\[ a: \text{“Carbon dioxide emissions are above the threshold } x \text{”} \]
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\[ a: \text{“Carbon dioxide emissions are above the threshold } x \text{”} \]
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\( a: \) “Carbon dioxide emissions are above the threshold \( x \)”

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Conclusion: Groups are inconsistent, difference between ’premise-based’ and ’conclusion-based’ decision making, ...
Discursive Dilemma

\(a: \text{“Carbon dioxide emissions are above the threshold } \times\text{”}\)

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\(b: \text{“There will be global warming”}\)

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Discursive Dilemma

*a:* “Carbon dioxide emissions are above the threshold $x$”

*a $\to$ b:* “If carbon dioxide emissions are above the threshold $x$, then there will be global warming”

*b:* “There will be global warming”

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Majority: True

Conclusion: Groups are inconsistent, difference between ‘premise-based’ and ‘conclusion-based’ decision making, ...
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**Conclusion**: Groups are inconsistent, difference between ‘premise-based’ and ‘conclusion-based’ decision making, ...
Group Preference Logics


Basic Ingredients

✓ informational attitudes (knowledge, group knowledge, belief, certainty, etc.)

✓ time, actions and ability (individual and coalitional ability)

✓ motivational attitudes (individual preferences, group preferences)
Once a semantics and language are fixed, then standard questions can be asked: eg. develop a proof theory, completeness, decidability, model checking.
How should we compare the different logical systems?

- Embedding one logic in another:

  - Coalition logic is a fragment of ATL (coalition logic).
  - Compare different models for a fixed language:
  - Comparing different frameworks:
    - eg. PDL vs. Temporal Logic, PDL vs. STIT, STIT vs. ATL, etc.
How should we compare the different logical systems?

- Embedding one logic in another: coalition logic is a fragment of ATL ($tr([C]\varphi) = \langle C \rangle \bowtie \varphi$)
General Issues

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How should we *merge* the different logical systems?
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- Combining logics is hard!

General Issues

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- Combining logics is hard!


**Theorem** \( \square \varphi \leftrightarrow \varphi \) is provable in combinations of Epistemic Logics and PDL with certain “cross axioms” \( (\square[a] \varphi \leftrightarrow [a] \square \varphi) \) (and full substitution).

Merging logics of rational agency

- Reasoning about information change (knowledge and time/actions)
- Knowledge, beliefs and certainty
- “Epistemizing” logics of action and ability: knowing how to achieve $\varphi$ vs. knowing that you can achieve $\varphi$
- Entangling knowledge and preferences
- Planning/intentions (BDI)
Merging logics of rational agency

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Conclusions
Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann’s problem: have a (trusted) friend tell Bob the time and subject of her talk.

Is this procedure correct?
Example

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There is a very simple procedure to solve Ann’s problem: have a (trusted) friend tell Bob the time and subject of her talk.

Is this procedure correct? Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob does not know that Ann knows that he knows about the talk.
5. And nothing else.
Example

$P$ means “The talk is at 2PM”.
Example

$P$ means “The talk is at 2PM”.

$\mathcal{M}, s \models K_A P \land \neg K_B P$
Example

\[ P \] means “The talk is at 2PM”.

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Example
Two Methodologies

**ETL methodology:** when describing a social situation, first write down all possible sequences of events, then at each moment write down the agents’ uncertainty, from that infer how the agents’ knowledge changes from one moment to the next.
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*Dynamic Epistemic Logic*

General Issues

The ‘Playground’

$t = 0$

$t = 1$

$t = 2$

$t = 3$
General Issues

The ‘Playground’
Formal Languages

- $P\varphi$ ($\varphi$ is true *sometime* in the past),
- $F\varphi$ ($\varphi$ is true *sometime* in the future),
- $Y\varphi$ ($\varphi$ is true at the previous moment),
- $N\varphi$ ($\varphi$ is true at the next moment),
- $N_e\varphi$ ($\varphi$ is true after event $e$),
- $K_i\varphi$ (agent $i$ knows $\varphi$) and
- $C_B\varphi$ (the group $B \subseteq A$ commonly knows $\varphi$).
History-based Models

An ETL model is a structure $\langle \mathcal{H}, \{\sim_i\}_{i \in A}, V \rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i \in A} \rangle$ is an ETL frame and $V : \text{At} \to 2^{\text{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs $H, t$:

$$H, t \models \varphi$$
Truth in a Model

- \( H, t \models P\varphi \iff \text{there exists } t' \leq t \text{ such that } H, t' \models \varphi \)
- \( H, t \models F\varphi \iff \text{there exists } t' \geq t \text{ such that } H, t' \models \varphi \)
- \( H, t \models N\varphi \iff H, t + 1 \models \varphi \)
- \( H, t \models Y\varphi \iff t > 1 \text{ and } H, t - 1 \models \varphi \)
- \( H, t \models K_i\varphi \iff \text{for each } H' \in \mathcal{H} \text{ and } m \geq 0 \text{ if } H_t \sim_i H'_m \text{ then } H', m \models \varphi \)
- \( H, t \models C\varphi \iff \text{for each } H' \in \mathcal{H} \text{ and } m \geq 0 \text{ if } H_t \sim_\ast H'_m \text{ then } H', m \models \varphi \).

Where \( \sim_\ast \) is the reflexive transitive closure of the union of the \( \sim_i \).
Truth in a Model

- \( H, t \models P\varphi \) iff there exists \( t' \leq t \) such that \( H, t' \models \varphi \)
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- \( H, t \models K_i \varphi \) iff for each \( H' \in \mathcal{H} \) and \( m \geq 0 \) if \( H_t \sim_i H'_m \) then \( H', m \models \varphi \)
- \( H, t \models C\varphi \) iff for each \( H' \in \mathcal{H} \) and \( m \geq 0 \) if \( H_t \sim_* H'_m \) then \( H', m \models \varphi \).

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General Issues
Returning to the Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.
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There is a very simple procedure to solve Ann’s problem: have a (trusted) friend tell Bob the time and subject of her talk.

Is this procedure correct?
$t = 0$

$H, 3 |= \varphi$
Bob’s uncertainty: $H, 3 \models \neg K_B P_{2PM}$
Bob’s uncertainty + ‘Protocol information’: $H, 3 |\equiv | K_B P_{2PM}$
Bob’s uncertainty + ‘Protocol information’:
\[ H, 3 \models \neg K_B K_A K_B P_{2PM} \]
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Parameters of the Logical Framework

1. Expressivity of the formal language. Does the language include a common knowledge operator? A future operator? Both?

2. Structural conditions on the underlying event structure. Do we restrict to protocol frames (finitely branching trees)? Finitely branching forests? Or, arbitrary ETL frames?

3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do the agents know what time it is?
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3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents’ know what time it is?
Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.

- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.

- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\text{len}(H) = \text{len}(H')$. 
Perfect Recall

$t = 0$

e_2 \quad e_4

e_1

t = 1

e_1 \quad e_5 \quad e_2 \quad e_3

e_1 \quad e_3 \quad e_7

t = 2

e_1 \quad e_3 \quad e_7 \quad e_6

t = 3

e_1 \quad e_1 \quad e_3
Perfect Recall

t = 0

\(e_1\) \(e_2\) \(e_4\)

\(t = 1\)

\(e_1\) \(e_5\) \(e_2\) \(e_3\)

\(t = 2\)

\(e_1\) \(e_3\) \(e_7\) \(e_6\)

\(t = 3\)

\(e_1\) \(e_3\)
Perfect Recall

t = 0

e_2  e_4

e_1  e_5

t = 1

e_1  e_5  e_2  e_3

e_1  e_3  e_7

t = 2

e_1  e_3  e_7  e_6

e_1  e_3

t = 3

e_1  e_2  e_7

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No Miracles

$t = 0$

$t = 1$

$t = 2$

$t = 3$
No Miracles

$t = 0$:

$t = 1$:

$t = 2$:

$t = 3$:
No Miracles
Ideal Agents

Assume there are two agents

Theorem
The logic of ideal agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).


Two Methodologies

ETL methodology: when describing a social situation, first write down all possible sequences of events, then at each moment write down the agents’ uncertainty, from that infer how the agents’ knowledge changes from one moment to the next.

Alternative methodology: describe an initial situations, provide a method for how events change a model that can be described in the formal language, then construct the event tree as needed.

Dynamic Epistemic Logic
Returning to the Example: DEL
Returning to the Example: DEL

\((\mathcal{M} \otimes E_1) \otimes E_2\)
Returning to the Example: DEL

\[(\mathcal{M} \otimes E_1) \otimes E_2\]

The initial model (Ann and Bob are ignorant about \(P_{2PM}\)).

Private announcement to Ann about the talk.
Abstract Description of the Event

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.
Recall the Ann and Bob example: **Charles tells Bob that the talk is at 2PM.**

![Diagram showing the flow of information between Ann, Bob, and the talk event.]

**Ann** knows which event took place.
Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

Ann knows which event took place.
Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

Bob thinks a different event took place.
Abstract Description of the Event

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

That is, Bob learns the time of the talk, but Ann learns nothing.
General Issues

Product Update

\[
\begin{align*}
\text{(s, e₁)} & \quad P \\
\text{(s, e₃)} & \quad P \\
\text{(t, e₃)} & \quad \neg P
\end{align*}
\]

\[
\begin{align*}
P & \quad \neg P \\
\text{e₁} & \quad P \\
\text{e₂} & \quad P \\
\text{e₃} & \quad T
\end{align*}
\]
Product Update

\[
(s, e_1) \quad \neg P \\
P \quad (s, e_2) \\
(s, e_3) \\
\neg P \quad (t, e_3)
\]
General Issues

Product Update

\[(s, e_1) \models \neg K_B K_A K_B P\]

\[(s, e_1) \models P\]

\[P \models (s, e_2)\]

\[(s, e_3) \models P\]

\[\neg P \models (t, e_3)\]
General Issues

Product Update

\[(s, e_1) \models \neg K_B K_A K_B P\]

\[(s, e_1) \rightarrow P \rightarrow P (s, e_2)\]

\[(s, e_3) \rightarrow P \rightarrow \neg P (t, e_3)\]
General Issues

Product Update

\((s, e_1) \models \neg K_B K_A K_B P\)  
\((s, e_1) \rightarrow P \rightarrow P \rightarrow (s, e_2)\)  
\((s, e_3) \rightarrow P \rightarrow \neg P \rightarrow (t, e_3)\)
General Issues

Product Update

\[ (s, e_1) \models \neg K_B K_A K_B P \]

\[ (s, e_1) \quad P \quad B \quad P \quad (s, e_2) \]

\[ (s, e_3) \quad P \quad B \quad \neg P \quad (t, e_3) \]
Let $\mathbb{M} = \langle W, R, V \rangle$ be a Kripke model.

An event model is a tuple $\mathbb{A} = \langle A, S, \text{Pre} \rangle$, where $S \subseteq A \times A$ and $\text{Pre} : \mathcal{L} \rightarrow \wp(A)$. 
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The update model $\mathcal{M} \otimes \mathcal{A} = \langle W', R', V' \rangle$ where
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- $(w, a)R'(w', a')$ iff $wRw'$ and $aSa'$
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- $(w, a) \in V(p)$ iff $w \in V(p)$

$\mathcal{M}, w \models [A, a] \varphi$ iff $\mathcal{M}, w \models Pre(a)$ implies $\mathcal{M} \otimes \mathcal{A}, (w, a) \models \varphi$. 

General Issues

Some Questions

▶ How do we relate the ETL-style analysis with the DEL-style analysis?
▶ In the DEL setting, what are the underlying assumptions about the reasoning abilities of the agents?
▶ Can we axiomatize interesting subclasses of ETL frames?

**Observation:** By repeatedly updating an epistemic model with event models, the machinery of DEL creates ETL models.
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Let $M$ be an epistemic model, and $P$ a DEL protocol (tree of event models). The ETL model generated by $M$ and $P$, forest($M$, $P$), represents all possible evolutions of the system obtained by updating $M$ with sequences from $P$. 
Example: Initial Model and Protocol

\[
\begin{align*}
&\text{Example: Initial Model and Protocol} \\
&s \quad \begin{array}{c} P, Q \end{array} \\
&t \quad \begin{array}{c} P, Q, R \end{array} \\
&v \quad \begin{array}{c} Q, R \end{array} \\
 & u \quad \begin{array}{c} P, R \end{array}
\end{align*}
\]
Example

\[ \begin{array}{c}
p_i, q_i, r_i \\
\downarrow \quad \downarrow \\
p, q, r \\
\downarrow \quad \downarrow \\
p, q, r \\
\downarrow \quad \downarrow \\
p, q, r \\
\downarrow \quad \downarrow \\
p, q, r \\
\end{array} \]
Example
General Issues

Example

```
(s) !P !P !P !P !P !P
(s, !P) !Q !Q !Q !Q !Q !Q
(s, !P, !Q) (t, !P, !Q) (t, !P, !Q) (u, !P, !R) (u, !P, !R) (v)
```

Diagram:

- States: 
  - s: P, Q
  - t: P, Q, R
  - u: P, R
  - v: Q, R

- Transitions:
  - i: s → v
  - j: t → u
  - j: t → v

- Worlds:
  - !P
  - !Q
  - !R
General Issues

Example
Example

\[
(s) \quad \text{(!P)} \quad (t) \quad \text{(!P)} \quad (u) \quad \text{(!P)} \\
(s, !P) \quad \text{(!Q)} \quad (t, !P) \quad \text{(!Q)} \quad (u, !P) \quad \text{(!Q)} \\
(s, !P, !Q) \quad \text{(!R)} \quad (t, !P, !Q) \quad \text{(!R)} \quad (u, !P, !R) \quad \text{(!R)}
\]
Example
Example
Example
Example
Given a set of DEL protocols $\mathbf{X}$, let $\mathcal{F}(\mathbf{X})$ be the class of ETL frames generated by protocols from $\mathbf{X}$.

**Theorem (Main Representation Theorem)**

Let $\Sigma$ be a finite set of events and suppose $\mathbf{X}_{\text{uni DEL}}$ is the class of uniform DEL protocols (with a finiteness condition). A model is in $\mathcal{F}(\mathbf{X}_{\text{uni DEL}})$ iff it satisfies propositional stability, synchronicity, perfect recall, local no miracles, and local bisimulation invariance.
Bisimulation Invariance + Finiteness Condition

t = 0

\[ \text{e}_2 \quad \text{e}_4 \]

\[ \text{e}_2 \quad \text{e}_1 \quad \text{e}_2 \]

\[ \text{e}_1 \quad \text{e}_5 \quad \text{e}_2 \quad \text{e}_1 \]

\[ \text{e}_1 \quad \text{e}_3 \quad \text{e}_7 \quad \text{e}_6 \]

\[ \text{e}_1 \quad \text{e}_1 \quad \text{e}_5 \]

\[ t = 1 \]

\[ t = 2 \]

\[ t = 3 \]

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General Issues

**Bisimulation Invariance + Finiteness Condition**

$t = 0$

$t = 1$

$t = 2$

$t = 3$
Recall that if $X$ is a set of DEL protocols, we define $F(X) = \{ F(M, P) \mid M \text{ an epistemic model and } P \in X \}$. This construction suggests the following natural questions:

▶ Which DEL protocols generate interesting ETL models?

▶ Which modal languages are most suitable to describe these models?

▶ Can we axiomatize interesting classes DEL-generated ETL models?

Announcement + Protocol Information

1. \( A \rightarrow \langle A \rangle^T \) vs. \( \langle A \rangle^T \rightarrow A \)
Announcement + Protocol Information

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2. \( \langle A \rangle K_i P \leftrightarrow A \land K_i \langle A \rangle P \)
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**Theorems** Sound and complete axiomatizations of various generated ETL models.
Merging logics of rational agency

- Reasoning about information change (knowledge and time/actions)
  - Knowledge, beliefs and certainty
  - “Epistemizing” logics of action and ability: knowing how to achieve $\varphi$ vs. knowing that you can achieve $\varphi$
- Entangling knowledge and preferences
- Planning/intentions (BDI)
Logics of Knowledge and Preference

\[ K(\varphi \succeq \psi) : \text{“Ann knows that } \varphi \text{ is at least as good as } \psi \text{”} \]

\[ K\varphi \succeq K\psi : \text{“knowing } \varphi \text{ is at least as good as knowing } \psi \text{”} \]
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\( \mathcal{M} = \langle W, \sim, \succeq, V \rangle \)
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J. van Eijck. Yet more modal logics of preference change and belief revision. manuscript, 2009.

$A(\psi \to \langle \preceq \rangle \varphi)$ vs. $K(\psi \to \langle \preceq \rangle \varphi)$
$A(\psi \rightarrow \langle \preceq \rangle \varphi) \text{ vs. } K(\psi \rightarrow \langle \preceq \rangle \varphi)$

**Should preferences be restricted to information sets?**
$A(\psi \rightarrow \langle \succeq \rangle \varphi)$ vs. $K(\psi \rightarrow \langle \succeq \rangle \varphi)$

Should preferences be restricted to information sets?

$\mathcal{M}, w \models \langle \succeq \cap \sim \rangle \varphi$ iff there is a $v$ with $w \sim v$ and $w \preceq v$ such that $\mathcal{M}, v \models \varphi$

$K(\psi \rightarrow \langle \succeq \cap \sim \rangle \varphi)$
Defining Beliefs from Preferences

- Starting with the work of Savage (based on Ramsey and de Finetti), there is a tradition in game theory and decision theory to define beliefs and utilities in terms of the agent’s preferences.
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Typically the results come in the form of a representation theorem:

*If the agents preferences satisfy such-and-such properties, then there is a set of conditional probability functions and a (state independent) utility function such that the agent can be assumed to act as an expected utility maximizer.*
Thus logical properties of beliefs can be derived from properties of preferences.

Let $\Omega$ be a set of states.

An **act** is a function $x : \Omega \to \mathbb{R}$. Let $\mathcal{A}^\Omega$ be the set of all acts.

$x_w$ for $w \in \Omega$ means that if the true state is $w$, then the agent receives prize $x$.

We write $x \succeq_w y$ the agent prefers $x$ over $y$ *provided the true state is* $w$. 
A belief operator is a function $B : 2^{\Omega} \rightarrow 2^{\Omega}$.

For $E \subseteq \Omega$, $w \in B(E)$ means the agent believes $E$ at state $w$.

$B$ is normal if

- $B(\Omega) = \Omega$
- $B(E \cap F) = B(E) \cap B(F)$

Possibility function: $P : \Omega \rightarrow 2^{\Omega}$: set of states the agent considers possible at $w$. 
Defining Beliefs from Preferences

For $E \subseteq \Omega$ and two acts $x$ and $y$, let $(x_E, y_{-E})$ denote the new act that is $x$ on $E$ and $y$ on $-E$. 
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$B$ reflects $\{\succeq_w\}_{w \in \Omega}$ provided for each $E \subseteq \Omega$

$$B(E) = \{w \mid (x_E, y_{-E}) \sim_w (x_E, z_{-E}) \text{ for all } x, y, z \in \mathcal{P}\Omega\}$$
**Theorem** If the preference relations are complete and transitive, then the derived belief operator is normal.

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Stemming from Bratman’s planning theory of intention a number of *BDI logics*:

- Cohen and Levesque; Rao and Georgeff; Meyer, van der Hoek (KARO); and many others.
Some Literature

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Some common features

- Underlying temporal model
- Belief, Desire, Intention, Plans, Actions are defined with corresponding operators in a language

Bratman’s Planning Theory of Intention

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A plan is a *conduct-controlling* mental attitude
Bratman’s Planning Theory of Intention


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An intention is a component of a future-directed plan.
Bratman’s Planning Theory of Intention

An agent commits to a (partial) plan that is

1. means-end coherent,
2. consistent with the agent's current beliefs and
3. stable (i.e., plans normally resist reconsideration)

"an agent's habits and dispositions concerning the reconsideration or nonreconsideration of a prior intention or plan determine the stability of that intention or plan". Furthermore, "The stability of [the agent's] plans will generally not be an isolated feature of those plans but will be linked to other features of [the agent's] psychology".
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Bratman’s Planning Theory of Intention

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Central to Bratman’s theory is the idea that these partial plans direct the agent’s deliberation by “constrain[ing] what options are considered relevant”:

“plans narrow the scope of the deliberation to a limited set of options. And they help to answer a question that tends to remain unanswered in traditional decision theory, namely: where do decision problems come from?”
A Methodological Issue

*What* are we formalizing? How will the logical framework be *used*?
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Two Extremes:

1. Formalizing a (philosophical) theory of rational agency:
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1. Formalizing a (philosophical) theory of rational agency: philosophers as intuition pumps generating "problems" for the logical frameworks.

2. Reasoning *about* multiagent systems.
A Methodological Issue

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Two Extremes:

1. Formalizing a (philosophical) theory of rational agency: philosophers as intuition pumps generating ”problems” for the logical frameworks.

2. Reasoning about multiagent systems. Three main applications of BDI logics: 1. a specification language for a MAS, 2. a programming language, and 3. verification language.

General Issues

C & L Logic of Intention

1. Intentions normally pose problems for the agent; the agent needs to determine a way to achieve them.

2. Intentions provide a “screen of admissibility” for adopting other intentions.

3. Agents “track” the success of their attempts to achieve their intentions.

4. If an agent intends to achieve \( p \), then
   
   4.1 The agent believes \( p \) is possible
   4.2 The agent does not believe he will not bring about \( p \)
   4.3 Under certain conditions, the agent believes he will bring about \( p \)
   4.4 Agents need not intend all the expected side-effects of their intentions.
C & L Logic of Intention

\[
\text{(PGOAL}_i p) := (\text{GOAL}_i(\text{LATER}p)) \land \\
(\text{BEL}_i \neg p) \land [\text{BEFORE}((\text{BEL}_i p) \lor (\text{BEL}_i \Box \neg p)) \neg (\text{GOAL}_i(\text{LATER}p))] \\
\text{(INTEND}_i a) := (\text{PGOAL}_i[\text{DONE}_i(\text{BEL}_i(\text{HAPPENS}a))]; a)]
\]
A third alternative:

3. Start from an explicit description of *what is being modeled*.
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Database/Planner Picture: Planner using a database to maintain its current set of beliefs.
General Issues

Planning vs. Database Management

1. How does an agent *generate* new intentions?

2. Given that the agent’s intentions specify a *partial plan*, how and when is the plan “filled out”?

3. How does an agent choose a particular *action* (that is under its control) given its current intentions?

4. How should an agent *maintain* its current state of beliefs and intentions in the presence of new information or new intentions?

5. When should an agent *reconsider* its intentions?
Our Framework

- What type of information does a planner provide? How do we represent a plan?

- Sources of beliefs

- Sources of dynamics: What can cause an agent’s database to change?

- Changing/amending plans vs. revising/updating beliefs
Elements of a Logic of Intention Revision
Beliefs in a dynamic environment: certainty (irrevocable knowledge, hard information), belief (revisable, soft information), safe belief
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Three views of actions: PDL (state changing), Temporal (lay out time and actions are sequences of time points), STIT (choices, or actions, constrain the future).
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- Three views of actions: PDL (state changing), Temporal (lay out time and actions are sequences of time points), STIT (choices, or actions, constrain the future).

- Two types of beliefs: those about the state of the world and those about the future which are governed by the agent’s plans
Intention Revision

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Practical reasoning rules: $\alpha \leftarrow \alpha_1, \alpha_2, \ldots, \alpha_n$

Intention Revision

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- Beliefs are sets of Linear Temporal Logic formulas (e.g., $\Box \varphi$)
- Desires are (possibly inconsistent) sets of Linear Temporal Logic formulas
- Practical reasoning rules: $\alpha \leftarrow \alpha_1, \alpha_2, \ldots, \alpha_n$
- Intentions are derived from the agents current active plans (trees of practical reasoning rules)
Many of the frameworks do discuss some form of intention revision.


- Two types of beliefs: strong beliefs vs. weak beliefs (beliefs that take into account the agent’s intentions)

- A dynamic update operator is defined ([Ω]ϕ)
Our Framework

1. *At a fixed moment*, a **choice situation** describes the current state-of-affairs (i.e., facts about the state-of-the-world), the tree of options that are available to the agent (i.e., the decision tree) and how actions change state of the world (i.e., the effect that performing an action will have on the state-of-the-world).
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2. At a fixed moment, a **model** describes the agent’s (current) beliefs (about the current state-of-the-world and what will become true in the future including options that will become available) and the agent's (current) instructions from the Planner (about future choices).
3. **Dynamic operators** representing each of the situations that may cause a change in beliefs and/or plans: learning a true fact, doing an action and receiving instructions from the Planner. These operators will describe how to relate models at different moments.
\[ M_w = (W, \{R_a\}_{a \in \text{Act}}, V, w) \]
General Issues

Choice Situations: $\mathcal{L}_1$

$$\varphi ::= p \mid \varphi \land \varphi \mid \neg \varphi \mid \langle a \rangle \varphi$$
General Issues

Choice Situations: $\mathcal{L}_1$

$\varphi := p \mid \varphi \land \varphi \mid \neg \varphi \mid \langle a \rangle \varphi$

- $M_w \models p$ iff $w \in V(p)$
- $M_w \models \varphi \land \psi$ iff $M_w \models \varphi$ and $M_w \models \psi$
- $M_w \models \neg \varphi$ iff $M_w \not\models \varphi$
- $M_w \models \langle a \rangle \varphi$ iff $\exists x \ wR_a x$ and $M_x \models \varphi$. 
Choice Situations: $L_1$

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**Notation:** If $\alpha = a_1 a_2 a_3 \cdots a_n$, $\langle \alpha \rangle \varphi := \langle a_1 \rangle \cdots \langle a_n \rangle \varphi$

$$N\varphi := \bigwedge_{a \in \text{Act}} [a] \varphi \quad [t] \varphi := \overbrace{N \cdots N}^{t \text{ times}} \varphi$$

$$P\varphi := \bigvee_{a \in \text{Act}} \langle a \rangle \varphi \quad \langle t \rangle \varphi := \overbrace{P \cdots P}^{t \text{ times}} \varphi$$
Adding Beliefs

Standard picture where worlds are choice situations
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Standard picture where worlds are choice situations

\[ \mathcal{M}_w \preceq \mathcal{N}_v : \text{Choice situation } \mathcal{N}_v \text{ is at least as plausible as } \mathcal{M}_w. \]
Adding Beliefs

Standard picture where worlds are choice situations

$$\mathcal{M}_w \preceq \mathcal{N}_v$$: Choice situation $\mathcal{N}_v$ is at least as plausible as $\mathcal{M}_w$.

1. Beliefs are about available options, current and future state of affairs: $Bp \land B\langle a \rangle \langle b \rangle \varphi$
2. Immediate options are known.
3. In the static model, restrict the language to only talk about current beliefs: $\langle a \rangle B\varphi$ is not well-formed
Belief Structures

\[ \mathcal{B} = (S, \preceq, M_w) \]
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Belief Structures

\( \mathcal{B} = (S, \preceq, M_w) \)
General Issues

Belief Structures

Language $(\mathcal{L}_2)$: $\varphi := \chi \mid \varphi \land \varphi \mid \neg \varphi \mid B(\varphi), \quad \chi \in \mathcal{L}_1$

Structures $\mathcal{B} = (S, \preceq, M_w)$ is a belief structure if:

(i) $S$ a set of choice situations
(ii) $\preceq$ is a plausibility ordering (reflexive, transitive, well-founded)
(iii) $M_w \in S$. 

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(iv) If $wRa x$ for some $x$ in $M$, then for all $N_v \in S$ s.t. $M_w \preceq N_v$, there is some $x'$ for which $vRa x'$ in $N$.

(v) If $M_w \preceq N_v$ and $vRa x$ for some $x$ in $N$, there is some $x' \in W$ such that $wRa x'$ in $M$. 
**Belief Structures**

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(v) If $\mathcal{M}_w \preceq \mathcal{N}_v$ and $vR_a x$ for some $x$ in $\mathcal{N}$, there is some $x' \in W$ such that $wR_a x'$ in $\mathcal{M}$. 
Belief Structures

\[ B \models \chi, \text{ iff } M_w \models \chi. \]
\[ B \models \varphi \land \psi, \text{ iff } B \models \varphi, \text{ and } B \models \psi. \]
\[ B \models \neg \varphi, \text{ iff } B \not\models \varphi. \]
\[ B \models B(\varphi), \text{ iff } \text{ for all } N_v \in Min_{\leq}(S), \ B, N_v \models \varphi. \]
Completeness

1. Standard proof works for the class of choice situations

2. The class of belief structures is also easily axiomatized ($\Box \varphi$ means $\varphi$ is true in all worlds at least as plausible as the current world):
   - **KD45** for $B$
   - $\langle a \rangle \top \rightarrow \Box (\langle a \rangle \top)$
   - $\Diamond (\langle a \rangle \top) \rightarrow \langle a \rangle \top$
Instructions

At each moment there are *instructions* from the Planner: We assume that at each moment, there are some instructions about future choices that the agent has agreed to follow (if he can).
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1. A complete plan, for each moment the specific action $a \in \text{Act}$ the agent will perform.
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4. Rather than instructing the agent to follow a specific (partial, conditional) plan, the Planner simply restricts the choices that are available to the agent in the future.
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4. Rather than instructing the agent to follow a specific (partial, conditional) plan, the Planner simply restricts the choices that are available to the agent in the future.
5. The Planner may provide a more complicated structure (subplan structure, goals, etc.)
Dynamics

There are three sources of dynamics:

1. Nature can reveal (true) facts about the current choice situation (e.g., facts that are true, choices that are available/not available in the future).
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We assume that only doing an action moves time forward. However, all three types of events may change the agent’s beliefs and current instructions.
$l = \{(b, i + 1), (d, i + 2)\}$
\[ I = \{(b, i + 1), (d, i + 2)\} \]

Add \((f, i + 3)\)
Say a set of beliefs $\mathcal{B}$ and a set of instructions $I$ is **coherent** if the agent doesn’t believe the instructions are impossible.

A **selection function** $\gamma$ maps a set of beliefs $\mathcal{B}$ and instructions to a set of instructions: $\gamma(\mathcal{B}, I) = I'$

1. $\gamma(\mathcal{B}, I) \subseteq I$.
2. $\gamma(\mathcal{B}, I)$ is coherent with $\mathcal{B}$.
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3. additional principles.....
AGM-style principles and representation theorem; Modal-style completeness (with dynamic operators get considerably more technical: *reduction axioms* are not available).

Moving to complex plans (with choice, concatenation and test):

1. The notion of Belief-Plan consistency must be updated
2. Define intentions semantically: the agent "intends $a$, $t$ just in case it is a necessary component of the current plan".
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What do the logical frameworks contribute to the discussion on rational agency?

- Normative vs. Descriptive
- Refine and test our intuitions: provide many answers to the question what is a rational agent?
- (epistemic) foundations of game theory

Logic and Game Theory, not Logic in place of Game Theory.
Conclusions

We are interested in reasoning about rational agents interacting in social situations.

What do the logical frameworks contribute to the discussion on rational agency?

▶ Normative vs. Descriptive
▶ refine and test our intuitions: provide many answers to the question what is a rational agent?
▶ (epistemic) foundations of game theory
Logic and Game Theory, not Logic in place of Game Theory.
▶ Social Software: Verify properties of social procedures
  • Refine existing social procedures or suggest new ones

Many types of informational attitudes: “hard” knowledge, belief, belief about the future state of affairs, “intention” based beliefs, revisable beliefs, safe beliefs.

Where does the “protocol” come from? What do the agents know about the protocol?
Conclusions

Logics of Rational Agency

- What’s going on in the area:
  www.loriweb.org

  (eds. T. Agotnes, J. van Benthem and EP)

- New subarea of *Stanford Encyclopedia of Philosophy* on logic and rational agency
  (eds. J. van Benthem, EP, and O. Roy)
Conclusions

Calls for....


► **Ph.D. position**: TiLPS, Tilburg University, “A formal analysis of social procedures”. Deadline: **October 15** (to start in February).
Thank You!